RHIC PROJECT

Brookhaven National Laboratory

Tolerance on $\Delta\theta$ Fluctuations in the Dipole

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The fluctuation of $\Delta\theta$ along the dipole increases the effective $\Delta\theta$ to be used in computing the closed orbit effect. A result for the effective rms $\Delta\theta$ is

$$\Delta\theta^2_{ef} = \Delta\theta^2_{av} + \Delta\theta^2_f \left(\Delta(\sqrt{\beta})/2\sqrt{\beta_c}\right)^2 \tag{1}$$

$\Delta(\sqrt{\beta})$ is the change in $\sqrt{\beta}$ over L/2; L is the dipole length. $\Delta(\sqrt{\beta}) = 1.5 \text{ m}^{1/2}$ in RHIC. β_c is β at the dipole center.

$\Delta\theta_{av}$ is the rms average $\Delta\theta$ in the dipole.

$\Delta\theta_f$ is the rms amplitude of the $\Delta\theta$ fluctuation around the average $\Delta\theta$.
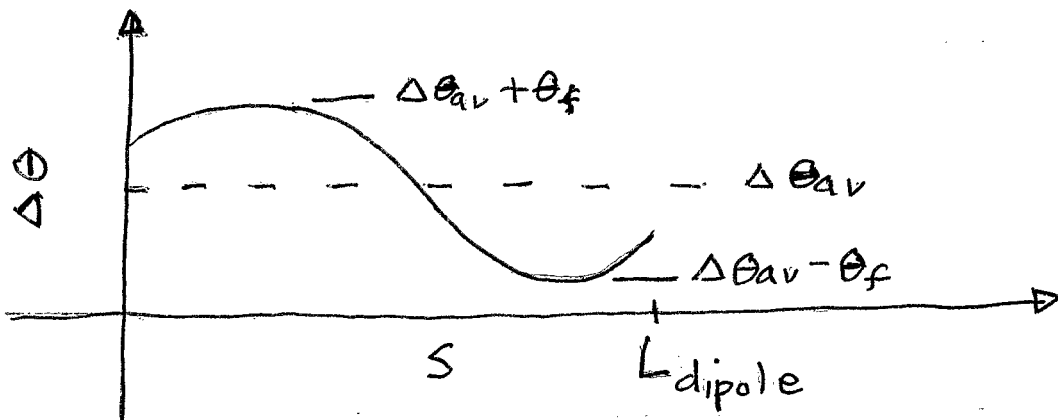
For RHIC dipoles, above gives

$$\Delta\theta^2_{ef} = \Delta\theta^2_{av} + (0.15 \ \Delta\theta_f)^2$$

For $\Delta\theta_f = 2$ mr, rms, the $\Delta\theta_{ef}$ is increased from $\Delta\theta_{ef} = 0.5$ mr rms to $\Delta\theta_{ef} = 0.58$, a 17% increase. The increase in the overall $\Delta\theta_{ef}$, including the survey error of 0.5 mr rms, is 10%.

$\Delta\theta_f = 2$ mr rms may be a reasonable choice for a tolerance on $\Delta\theta_f$.

The above results assume a model where $\Delta\theta$ along the dipole is as shown below:

Note, the tolerance on $\Delta\theta_{av}$ would still be 0.5 mr rms.

The closed orbit error with this model can be computed from

$$\Delta y \sim \sum_{dipoles} \int ds \ \Delta\theta \ g(s)$$

$$g(s) = \sqrt{\beta} \ \cos(\pi\nu - (\psi - \psi_o))$$

$$\Delta\theta = \Delta\theta_{av} + \Delta\theta_f \ f(s)$$

Assuming that $\Delta\theta_{av}$ and $\Delta\theta_f$ vary randomly from dipole to dipole, and $f(s)$ has the shape in the above figure, then one derives the above result for the rms effective $\Delta\theta$.

A more accurate result than Eq. (1) for $\Delta\theta_{ef}$, which includes the effect of the variation in the betatron phase over the dipole, is the following

$$\Delta\theta^2_{ef} = \Delta\theta^2_{av} + \Delta\theta^2_f \left[\left(\frac{\Delta(\sqrt{\beta})}{2\sqrt{\beta}_c}\right)^2 + \left(\frac{L}{4\beta_c}\right)^2 \right]$$

The added term due to the phase variation can usually be neglected.